**Question 1:**

**Following the method of fractions enumerations discussed in the class, please explain, what order number in the enumeration would be assigned to 6/7? Consider a general case: explain a formula for assigning an order number in the enumeration for fraction m/n.**

Let’s consider a fractional matrix:

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 1/1 | 1/2 | 1/3 | ¼ | 1/5 | 1/6 | 1/7 | 1/8 | 1/9 | 1/10 | 1/11 | 1/12 | …… |
| 2/1 | 2/2 | 2/3 | 2/4 | 2/5 | 2/6 | 2/7 | 2/8 | 2/9 | 2/10 | 2/11 | 2/12 | …… |
| 3/1 | 3/2 | 3/3 | ¾ | 3/5 | 3/6 | 3/7 | 3/8 | 3/9 | 3/10 | 3/11 | 3/12 | …… |
| 4/1 | 4/2 | 4/3 | 4/4 | 4/5 | 4/6 | 4/7 | 4/8 | 4/9 | 4/10 | 4/11 | 4/12 | …… |
| 5/1 | 5/2 | 5/3 | 5/4 | 5/5 | 5/6 | 5/7 | 5/8 | 5/9 | 5/10 | 5/11 | 5/12 | …… |
| 6/1 | 6/2 | 6/3 | 6/4 | 6/5 | 6/6 | 6/7 | 6/8 | 6/9 | 6/10 | 6/11 | 6/12 | …… |
| 7/1 | 7/2 | 7/3 | 7/4 | 7/5 | 7/6 | 7/7 | 7/8 | 7/9 | 7/10 | 7/11 | 7/12 | …… |
| 8/1 | 8/2 | 8/3 | 8/4 | 8/5 | 8/6 | 8/7 | 8/8 | 8/9 | 8/10 | 8/11 | 8/12 | …… |
| 9/1 | 9/2 | 9/3 | 9/4 | 9/5 | 9/6 | 9/7 | 9/8 | 9/9 | 9/10 | 9/11 | 9/12 | …… |
| 10/1 | 10/2 | 10/3 | 10/4 | 10/5 | 10/6 | 10/7 | 10/8 | 10/9 | 10/10 | 10/11 | 10/12 | …… |
| 11/1 | 11/2 | 11/3 | 11/4 | 11/5 | 11/6 | 11/7 | 11/8 | 11/9 | 11/10 | 11/11 | 11/12 | …… |
| 12/1 | 12/2 | 12/3 | 12/4 | 12/5 | 12/6 | 12/7 | 12/8 | 12/9 | 12/10 | 12/11 | 12/12 | …… |
| .  .  .  . | .  .  .  . | .  .  .  . | .  .  .  . | .  .  .  . | .  .  .  . | .  .  .  . | .  .  .  . | .  .  .  . | .  .  .  . | .  .  .  . | .  .  .  . | .……  .……  .……  .…… |

We always count from top diagonally as represented above:

Diagonal number of 6/7 = (6+7) – 1 = 13 - 1 = 12.

To get the total number of fractions present in previous 11 diagonal = 11\*12/2 = 66

So, numeration number for 6/7 = 66 + 6 = 72.

For general case, formula for assigning an order number in the enumeration for fraction m/n:

Diagonal number = (m+n)-1

Total number of fraction = ((((m+n)-1)\*((m+n)-2))/2)+m.

**Question 2:**

**Please prove that the set of all infinite sequences of 1’s and 0’s is not countable. Suppose somebody claims that he can enumerate infinite sequence of 1’s and 0’s and that is how his enumeration starts**

**11111…**

**00000…**

**01010…**

**00000…**

**What would be the first four digits of the counter example sequence?**

Suppose there exists an enumeration of all infinite sequences of 1’s and 0’s. in order to get a contradiction with this statement, we will construct a counter example sequence, that would not be in this enumeration which would contradict the claim that all infinite sequence of 1’s and 0’s are enumerated.

Construction of counterexample sequence α: for the first digit of α, take Boolean complement of the first digit of the first sequence. For the second digit of α, take the Boolean complement of the second digit of the second sequence. For the third digit of α, take Boolean complement of the third digit of the third sequence and so on. For the Nth digit of α, take Boolean complement of the Nth digit of the Nth sequence. Thus, α is different from any sequence in the suggested enumeration: it differs from the first sequence of the first digit, it differs from the second sequence because of the second digit, and for any n, it differs from n-th sequence because of the n-th digit.

11111…

00000…

01010…

00000…

In order to construct the first four digits of α we take the complements of the underlined 1 or 0.

Thus, the first four digit of α are 0111.

**4.Question:**

**Suppose an infinite hotel with countably many rooms is empty. Infinitely many(countably) infinite buses (with countably many seats) are coming to town. Please provide an algorithm to accommodate all passengers in the hotel.**

If we needed to accommodate room for 5 guests, then we move everyone in the hotel up 5 rooms, leaving the first 5 empty. And when we needed to make room for a countably infinite set of guests, we moved all our current guests to the even numbered rooms, leaving an infinite number of odd numbered rooms for our new guests.

So, to do so which device can we appoint to move our current guests that will keep an infinite number of infinite rooms?

The set of primes is countably infinite, we can take the guests and systematically assign them to powers of prime numbers. Set of all prime numbers {2, 3, 5, 7, 11, 13, .....}.

Assigning guests of bus#1 to hotel rooms according to seat number of guests (i.e natural numbers) to the power of the first prime number, 2. So, the guest in seat number 1 goes to room 21 = 2, the guest in room 2 goes to room 22 = 4, and so on to 2n.

Bus #1 seat number room number

Now, let’s assign the infinite guests from bus #2 to rooms in hotel by taking the next prime number, 3. Using their seat number as the powers.

For example, in this first bus we will move the guest from seat 1 to 31 = 3, the guest from seat 2 to 32 = 9, and so on to 3n.

Bus #2 seat number room number

From the above analysis now to assign seats for bus #3 rooms for infinite number of guests to the hotel rooms. We will follow the same pattern using the next prime number as the base for the next bus.

So, for bus #3 seat number 1 then the room assigned is 51 = 5, for seat number 2 the room assigned is 52 = 25, and so on to 5n.

Bus #3 seat number room number

Let’s consider same method for all the infinite set of buses. Because we have an infinite set of prime numbers, theoretically we can continue using the next prime as the base for each of the countably infinite buses. And by increasing the prime number to a new power produces a brand-new room number, we can locate rooms for the infinite guests on each bus.

This way we can solve a path to ensure a different room for an infinite number of infinite guests.